

Mass Generation, Ghost Condensation and Broken Symmetry: $SU(2)$ in Covariant Abelian Gauges

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Abstract

The local action of an $SU(2)$ gauge theory in general covariant Abelian gauges and the associated equivariant BRST symmetry that guarantees the perturbative renormalizability of the model are given. A global $SL(2, \mathbb{R})$ symmetry of the model is spontaneously broken by ghost-antighost condensation at arbitrarily small coupling. This leads to propagators that are finite at Euclidean momenta for all elementary fields except the Abelian “photon”. Ward Identities show that the symmetry breaking gives rise to massless BRST-quartets with ghost numbers $(1, 2, -2, -1)$ and $(0, 1, -1, 0)$. The latter quartet is interpreted as due to an Abelian Higgs mechanism in the dual description of the model.

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1 Introduction

Although QCD is asymptotically free, the perturbative analysis of the high-temperature phase is plagued by infrared (IR) divergences¹. The best one can presently achieve perturbatively at high temperatures is a resummation of the infrared-safe contributions². The situation is somewhat embarrassing, since one naively might expect that the high temperature phase of an asymptotically free theory is perturbative. The IR-problem encountered in the perturbative high temperature expansion is part of the more general problem of defining a non-Abelian gauge-fixed theory on a compact Euclidean space-time without boundaries, such as a hypertorus. It was shown³ that normalizable ghost zero-modes in this case cause the partition function to vanish in conventional covariant gauges. An equivariant BRST construction was used to eliminate these ghost zero-modes associated with global gauge invariance at the expense of a non-local quartic ghost interaction. This interaction leads to ghost-antighost condensation at arbitrarily small coupling^{3,4}.

The equivariant gauge-fixing procedure later was used to reduce the structure group of an $SU(2)$ lattice gauge theory (LGT) to a physically equivalent

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Abelian LGT with a $U(1)$ structure group⁵. This lattice formulation is the only known definition of an non-Abelian (equivariant) BRST symmetry that is not just valid perturbatively. The associated local quartic ghost interaction leads to ghost-antighost condensation in this case as well. The starting point of this investigation is a transcription of the partially gauge-fixed $SU(2)$ -LGT to the continuum using the equivariant BRST algebra. Note that the global equivariant BRST-symmetry makes the construction of the critical continuum model of the partially gauge fixed non-Abelian LGT practically unique. This aspect of the equivariant BRST symmetry is of some interest in itself⁶.

2 The Model

In a Euclidean space-time, the critical continuum action of the lattice model⁵ is uniquely specified by the BRST algebra, the field content and power counting. Decomposing the non-Abelian $SU(2)$ connection $\vec{A}_\mu = (W_\mu^1, W_\mu^2, A_\mu)$ in terms of two real vector-bosons (or one complex one) and a $U(1)$ -connection A_μ (the “photon” of the model), the loop expansion is defined by the Lagrangian,

$$\mathcal{L} = \mathcal{L}_{\text{inv.}} + \mathcal{L}_{\text{AG}} + \mathcal{L}_{\text{aGF}} . \quad (1)$$

Here $\mathcal{L}_{\text{inv.}}$ is the usual $SU(2)$ -invariant Lagrangian written in terms of the vector bosons and the photon^a,

$$\mathcal{L}_{\text{inv.}} = \mathcal{L}_{\text{matter}} + \frac{1}{4}(G_{\mu\nu}G_{\mu\nu} + G_{\mu\nu}^a G_{\mu\nu}^a) , \quad (2)$$

with

$$\begin{aligned} G_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - g\varepsilon^{ab}W_\mu^a W_\nu^b \\ G_{\mu\nu}^a &= D_\mu^{ab}W_\nu^b - D_\nu^{ab}W_\mu^b \\ &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\varepsilon^{ab}(A_\mu W_\nu^b - A_\nu W_\mu^b) . \end{aligned} \quad (3)$$

\mathcal{L}_{AG} partially gauge-fixes to the maximal Abelian subgroup $U(1)$ of $SU(2)$ in a covariant manner,

$$\mathcal{L}_{\text{AG}} = b^a F^a - \frac{\alpha}{2}b^a b^a - \bar{c}^a M^{ab}c^b - g^2 \frac{\alpha}{2}(\bar{c}^a \varepsilon^{ab}c^b)^2 , \quad (4)$$

with

$$\begin{aligned} F^a &= D_\mu^{ab}W_\mu^b = \partial_\mu W_\mu^a + gA_\mu \varepsilon^{ab}W_\mu^b \\ M^{ab} &= D_\mu^{ac}D_\mu^{cb} + g^2 \varepsilon^{ac}\varepsilon^{bd}W_\mu^c W_\mu^d . \end{aligned} \quad (5)$$

^aLatin indices take values in $\{1, 2\}$ only, Einstein's summation convention applies and $\varepsilon^{12} = -\varepsilon^{21} = 1$, vanishing otherwise. All results are in the \overline{MS} renormalization scheme.

Note that $L_{U(1)} = L_{\text{inv.}} + \mathcal{L}_{\text{AG}}$ is invariant under $U(1)$ -gauge transformations *and* under an on-shell BRST symmetry s and anti-BRST symmetry \bar{s} , whose action on the fields is

$$\begin{aligned} sA_\mu &= g\varepsilon^{ab}c^aW_\mu^b & \bar{s}A_\mu &= g\varepsilon^{ab}\bar{c}^aW_\mu^b \\ sW_\mu^a &= D_\mu^{ab}c^b & \bar{s}W_\mu^a &= D_\mu^{ab}\bar{c}^b \\ sc^a &= 0 & \bar{s}\bar{c}^a &= 0 \\ s\bar{c}^a &= b^a & \bar{s}c^a &= -b^a \\ sb^a &= s^2\bar{c}^a = \frac{g}{2}\varepsilon^{ab}\bar{c}^b c^c \varepsilon^{cd}c^d & \bar{s}b^a &= -\frac{g}{2}\varepsilon^{ab}c^b \bar{c}^c \varepsilon^{cd}\bar{c}^d, \end{aligned} \quad (6)$$

with an obvious extension to include matter fields. Contrary to most other proposals for mass generation^{7,8} the BRST algebra Eq. (6) closes on-shell on the set of $U(1)$ -invariant functionals: on functionals that depend only on W, A, c and the matter fields, s^2 for instance effects an infinitesimal $U(1)$ -transformation with the parameter $\frac{g}{2}\varepsilon^{ab}c^a c^b$. The algebra Eq. (6) defines an equivariant cohomology. The lattice regularization of this symmetry⁵ ensures that the model is renormalizable and unitary. Perturbative renormalizability and unitarity has recently also been algebraically established for this model⁹. Note that the physical sector comprises states created by composite operators of A, W and the matter fields in the equivariant cohomology of s (or \bar{s}). They are BRST closed *and* $U(1)$ -invariant.

Expectation values of physical observables of the $U(1)$ -LGT are the same⁵ as those of the original $SU(2)$ -LGT for any $\alpha > 0$. Note that setting $\alpha = 0$ and formally solving the constraint¹⁴ $F^a = 0$ is *not* the same as taking the limit $\alpha \rightarrow 0$. The non-perturbative reason is exhibited by the lattice calculation⁵: without the quartic ghost interaction, Gribov copies of a configuration conspire to give *vanishing* expectation values for all physical observables. No matter how small, the quartic ghost interaction is necessary for a normalizable partition function and expectation values of physical observables that are identical with those of the original $SU(2)$ -LGT. Without Abelian ghosts, a quartic ghost interaction is generated by perturbative corrections even in the $\alpha \rightarrow 0$ limit¹⁰. Currently, a lattice regularization only exists for the continuum model described by Eq. (1) – with decoupled Abelian ghosts and an $SL(2, \mathbb{R})$ symmetry.

In the continuum model \mathcal{L}_{aGF} in Eq. (1) can be added “by hand” to fix the remaining $U(1)$ gauge invariance and define the perturbative series of the continuum model unambiguously. Note that this Abelian gauge fixing does not introduce new ghosts. I will assume a conventional covariant gauge-fixing term,

$$L_{\text{aGF}} = \frac{(\partial_\mu A_\mu)^2}{2\xi}. \quad (7)$$

None of the following conclusions depend on the gauge-fixing of the Abelian subgroup – they in particular do not depend on ξ .

3 Ghost Condensation and the Spontaneously Broken SL(2,R) Symmetry

The Lagrangian Eq. (1) is invariant under a global bosonic SL(2,R) symmetry generated by

$$\Pi^+ = \int c^a(x) \frac{\delta}{\delta \bar{c}^a(x)} , \quad \Pi^- = \int \bar{c}^a(x) \frac{\delta}{\delta c^a(x)} , \quad (8)$$

and the ghost number $\Pi = [\Pi^+, \Pi^-]$. This SL(2,R) symmetry is inherited from the lattice regularized model⁵ and therefore is not anomalous. The conserved currents corresponding to Π^\pm are U(1)-invariant and BRST, respectively anti-BRST exact,

$$j_\mu^+ = c^a D_\mu^{ab} c^b = s c^a W_\mu^a , \quad j_\mu^- = \bar{c}^a D_\mu^{ab} \bar{c}^b = \bar{s} \bar{c}^a W_\mu^a . \quad (9)$$

Because the currents Eq. (9) are (anti)-BRST exact, a spontaneously broken SL(2,R) symmetry is accompanied by a BRST-quartet of massless Goldstone states with ghost numbers 1, 2, -2 and -1. They are U(1)-invariant $c-W, c-c, \bar{c}-\bar{c}$ and $\bar{c}-W$ bound states. Such BRST quartets do not contribute to physical quantities¹⁵ like the free energy^b. The spontaneous symmetry breaking in this sense is similar to a (dynamical) Higgs mechanism in the adjoint.

An order parameter for the spontaneous breaking of the SL(2,R) symmetry is

$$\langle \bar{c}^a \varepsilon^{ab} c^b \rangle = \frac{1}{2} \langle \Pi^- (c^a \varepsilon^{ab} c^b) \rangle = -\frac{1}{2} \langle \Pi^+ (\bar{c}^a \varepsilon^{ab} \bar{c}^b) \rangle . \quad (10)$$

One can argue for such a ghost condensate in the background of a degenerate orbit such as that of a non-Abelian monopole. The solution to $F^a = 0$ in this case is not unique locally and the operator M^{ab} in such a background has an even number of ghost zero-modes that interact via the quartic ghost interaction only. Similar to states at the Fermi surface of a BCS-superconductor, condensation occurs in the attractive channel of the quartic interaction. The notion that the predominance of degenerate orbits may trigger ghost condensation can be sharpened considerably by considering the Ward identities associated with the broken SL(2,R) symmetry.

Using that the currents Eq. (9) of the broken $SL(2, R)$ symmetry are U(1)-invariant and (anti)-BRST exact, one can show that

$$\langle s(\bar{c}^a \varepsilon^{ab} \bar{c}^b)(0) \quad c^a W_\mu^a(x) \rangle = \frac{\langle \bar{c}^a \varepsilon^{ab} \bar{c}^b \rangle x_\mu}{\pi^2 x^4} = -\langle \bar{s}(c^a \varepsilon^{ab} c^b)(0) \quad \bar{c}^a W_\mu^a(x) \rangle , \quad (11)$$

^bThis is analogous to the decoupling of the BRST quartet of the weak interaction in R_ξ gauges¹⁵.

implying massless asymptotic states with ghost numbers ± 1 . Together with the Goldstone states with ghost number ± 2 these form the (anti)-BRST-quartets of the spontaneously broken $\text{SL}(2, \mathbb{R})$ symmetry.

However, the symmetries imply even more massless asymptotic states when the $\text{SL}(2, \mathbb{R})$ symmetry is spontaneously broken. Since,

$$\begin{aligned} s(\bar{c}^a \varepsilon^{ab} \bar{c}^b) &= 2b^a \varepsilon^{ab} \bar{c}^b = -2\bar{s}(c^a \varepsilon^{ab} \bar{c}^b) \\ \bar{s}(c^a \varepsilon^{ab} \bar{c}^b) &= -2b^a \varepsilon^{ab} \bar{c}^b = -2s(\bar{c}^a \varepsilon^{ab} \bar{c}^b) , \end{aligned} \quad (12)$$

the spontaneous symmetry breaking is associated with yet another (anti)-BRST-quartet of massless states whose ghost numbers are $0, 1, -1, 0$. Using Eq. (12) in Eq. (11) gives

$$-\langle \bar{s}(c^a \varepsilon^{ab} \bar{c}^b(0)) \quad c^a W_\mu^a(x) \rangle = \frac{\langle \bar{c}^a \varepsilon^{ab} \bar{c}^b \rangle x_\mu}{2\pi^2 x^4} = \langle s(\bar{c}^a \varepsilon^{ab} \bar{c}^b(0)) \quad \bar{c}^a W_\mu^a(x) \rangle . \quad (13)$$

There are thus asymptotic massless states with vanishing ghost number

$$\langle \bar{c}^a \varepsilon^{ab} \bar{c}^b(0) \quad \bar{s}(c^a W_\mu^a(x)) \rangle = \frac{\langle \bar{c}^a \varepsilon^{ab} \bar{c}^b \rangle x_\mu}{2\pi^2 x^4} = -\langle \bar{c}^a \varepsilon^{ab} \bar{c}^b(0) \quad s(\bar{c}^a W_\mu^a(x)) \rangle . \quad (14)$$

Eq. (14) is consistent with the absence of a massless pole in correlations of the ghost number current

$$j_\mu^0 = -\frac{1}{2}[s(\bar{c}^a W_\mu^a) + \bar{s}(c^a W_\mu^a)] = \frac{1}{2}[\bar{c}^a D^{ab} c^b + c^a D^{ab} \bar{c}^b] . \quad (15)$$

The BRST-quartet of massless states implied by Eq. (14) could also be the result of a spontaneously broken Abelian gauge symmetry. To see this, consider a $\text{U}(1)$ gauge model with a self-interacting charged scalar $\Phi(x)$ that transforms as $\delta\Phi(x) = i\Phi(x)\delta\theta(x)$, the Higgs mechanism leads to the relation

$$\left\langle \Phi(0) \quad \frac{\delta S}{\delta\theta(x)} \right\rangle = i\delta^4(x) \langle \Phi \rangle , \quad (16)$$

where S is the gauge-fixed action of this Abelian model. In covariant gauges that do not break the global $\text{U}(1)$ invariance, $\delta S/\delta\theta(x) = -i\partial_\mu J_\mu(x)$ is a BRST-exact current $J_\mu(x) = s\bar{C}_\mu(x)$. The fundamental BRST-quartet¹⁵ remains massless in gauges that do not break the global $\text{U}(1)$ invariance under consideration.

If we identify $\bar{c}^a W_\mu^a$ with the current \bar{C}_μ and $\bar{c}^a \varepsilon^{ab} \bar{c}^b$ with the charged Higgs scalar $\Phi(x)$ of the effective Abelian Higgs model, the right hand side of Eq. (14) is the analogue of Eq. (16).

Note that the spontaneously broken Abelian (gauge) symmetry in the present case has vanishing ghost number but cannot be the chromo-electric U(1) nor the ghost number of the covariant MAG: the scalar $\bar{c}^a \varepsilon^{ab} c^b$ is neutral under both. Eq. (14) implies that a massless asymptotic state with vanishing chromo-electric charge and ghost number is created by the longitudinal part of the composite vector field

$$B_\mu := b^a W_\mu^a - \partial_\mu (\bar{c}^a c^a) = \frac{1}{2} [s(\bar{c}^a W_\mu^a) - \bar{s}(c^a W_\mu^a)] . \quad (17)$$

The fact that B_μ does not mix with A_μ under renormalization supports the conjecture that the chromo-electric U(1) symmetry of MAG remains unbroken by ghost condensation in this channel. It is tempting to assume that the transverse part of B_μ has non-vanishing overlap with the *dual* photon of the model. Since the vector bosons are expected to be massive in the confining phase, this is not in contradiction with a dual Higgs mechanism¹¹.

The dynamical formation of a

$$\langle W_\mu^a W_\mu^a - \alpha \bar{c}^a c^a \rangle \quad (18)$$

condensate is discussed by Kondo¹². It generates effective masses for the vector bosons and ghosts without apparently breaking any continuous symmetries of the model. Contrary to the condensate of Eq. (10), the condensate of Eq. (18) is not easily linked to a dual Abelian Higgs mechanism and leads to power corrections of dimension 2 in physical correlation functions. [By contrast, the leading power corrections due to the condensate of Eq. (10) in physical correlation functions are of dimension 4.] The condensate of Eq. (18) therefore could be characteristic for the Coulomb phase¹¹ of the model. This interesting possibility will not be further pursued here.

4 Perturbations in the Broken Phase

To investigate the perturbative consequences of $\langle \bar{c}^a \varepsilon^{ab} c^b \rangle \neq 0$, the quartic ghost interaction in Eq. (4) is linearized with the help of an auxiliary scalar field $\rho(x)$ of canonical dimension two. Adding the quadratic term

$$\mathcal{L}_{\text{aux}} = \frac{1}{2g^2\alpha} (\rho/\sqrt{Z} - g^2\alpha\sqrt{Z}\bar{c}^a \varepsilon^{ab} c^b)^2 \quad (19)$$

to the Lagrangian of Eq. (1), the tree level quartic ghost interaction vanishes at $Z = 1$ and is then formally of $O(g^4)$, proportional to g^2 and $Z - 1$:^c

^c The discrete symmetry $c^a \rightarrow \bar{c}^a$, $\bar{c}^a \rightarrow -c^a$, $\rho \rightarrow -\rho$ relating s and \bar{s} also ensures that ρ only mixes with $\bar{c}^a \varepsilon^{ab} c^b$.

The perturbative expansion about a *non-trivial* solution $\langle \rho \rangle = v \neq 0$ to the gap equation

$$\frac{v}{g^2 \alpha} = \langle c^a(x) \varepsilon^{ab} \bar{c}^b(x) \rangle|_{\langle \rho \rangle = v} , \quad (20)$$

turns out to be well behaved in the infrared. Note that Eq. (20) is $U(1)$ -invariant and therefore does not depend on the $U(1)$ gauge-fixing Eq. (7). Let us for the moment assume that a unique non-trivial solution to Eq. (20) exists in some gauge α ; we return to this conjecture below. The consequences for the IR-behavior of the model are dramatic. Defining the quantum part $\sigma(x)$ of the auxiliary scalar ρ by

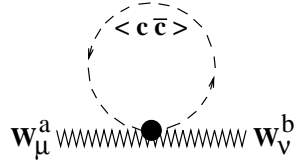
$$\rho(x) = v + \sqrt{\alpha} \sigma(x) \quad \text{with} \quad \langle \sigma \rangle = 0 , \quad (21)$$

the momentum representation of the Euclidean ghost propagator at tree level becomes

$$\langle c^a \bar{c}^b \rangle_p = \frac{p^2 \delta^{ab} + v \varepsilon^{ab}}{p^4 + v^2} . \quad (22)$$

Feynman's parameterization of this propagator allows an evaluation of loop integrals using dimensional regularization that is only slightly more complicated than usual. More importantly, the ghost propagator is regular at Euclidean momenta when $v \neq 0$. Its complex conjugate poles at $p^2 = \pm i v$ can furthermore not be interpreted as due to asymptotic ghost states¹⁶.

When $v \neq 0$, the W -boson is massless only at tree level and (see Fig. 1) acquires the *finite* mass $m_W^2 = g^2 |v| / (16\pi)$ at one loop,



$$= \frac{g^2 |v|}{16\pi} \delta_{\mu\nu} \delta^{ab} . \quad (23)$$

Fig. 1. The finite one-loop contribution to the W mass.

Technically, the one-loop contribution is finite because the integral in Eq. (23) involves only the δ^{ab} -part of the ghost propagator Eq. (22). Since $p^2/(p^4 + v^2) = -v^2/(p^2(p^4 + v^2)) + 1/p^2$, the v -dependence of the loop integral is IR- and UV-finite. The quadratic UV-divergence of the $1/p^2$ subtraction at $v = 0$ is cancelled by the other, v -independent, quadratically divergent one-loop contributions – (in dimensional regularization this scale-invariant integral vanishes by itself). m_W^2 furthermore is *positive* due to the overall minus sign of the *ghost* loop. The sign of m_W^2 is crucial, for it indicates that the model is *stable* and (as far as the loop expansion is concerned) does not develop tachyonic poles at Euclidean p^2 for $v \neq 0$. Conceptually, the local mass term proportional

to $\delta_{\mu\nu}\delta^{ab}$ is finite due to the BRST symmetry Eq. (6), which excludes a mass counter-term. The latter argument implies that contributions to m_W^2 are finite to all orders of the loop expansion.

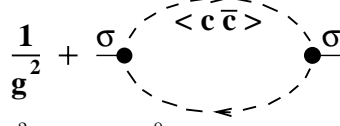


Fig. 2. $\Gamma_{\sigma\sigma}(v, p^2)$ to order g^0 .

If the model is stable at $v \neq 0$, the 1PI 2-point function $\Gamma_{\sigma\sigma}(v, p^2)$ of the scalar must not vanish at Euclidean p^2 either. To order g^0 , $\Gamma_{\sigma\sigma}(p^2)$ is given by the $1/g^2$ term that arises from Eq. (19) upon substitution of Eq. (21) and the one-(ghost)-loop contribution shown in Fig.2. Since a non-trivial solution to the gap equation Eq. (20) relates $1/g^2$ to a loop integral of zeroth order in the coupling, we may use Eq. (20) to lowest order to obtain a “tree-level” expression for $\Gamma_{\sigma\sigma}(v, p^2)$ of order g^0 . Evaluating the loop integrals, one obtains the real, positive and monotonic function

$$\Gamma_{\sigma\sigma}(x := \frac{\sqrt{\alpha v^2}}{p^2}) = \left\{ \frac{-1 + 2\sqrt{1 - 4ix} \operatorname{acoth}(\sqrt{1 - 4ix})}{32\pi^2\alpha^{-1}} \right\} + \{x \rightarrow -x\} . \quad (24)$$

$\Gamma_{\sigma\sigma}(p^2 \geq 0) \geq \alpha/(16\pi^2)$ to order g^0 establishes the perturbative stability of a non-trivial solution to Eq. (20) and the fact that this solution is a minimum of the one-loop effective potential.

An expansion about a solution $v \neq 0$ to the gap equation thus has lowest order propagators that are regular at Euclidean momenta for all the elementary fields except the photon A_μ (if all the matter fields are massive). The polarization of the photon vanishes at $p^2 = 0$ due to the $U(1)$ -symmetry – regardless of the value of v . Taking into account that the massless Goldstone quartets associated with this symmetry breaking decouple from physical quantities, the situation for $v \neq 0$ is thus rather similar to QED with an unorthodox massive matter content (extending the notion of “massive matter” to include ghosts and other unphysical fields).

5 The Gap

To complete the argument, we solve Eq. (20) for small coupling. To lowest order in the loop expansion, the relation between the renormalized couplings g, α , the renormalization point μ and an expectation value $v \neq 0$ implied by

Eq. (20) is

$$\ln \frac{v^2}{\mu^4} = -\frac{16\pi^2}{\alpha g^2} + 2 + O(g^2) . \quad (25)$$

The anomalous dimension γ_v of the expectation value is simultaneously found to be^d

$$\gamma_v = -\frac{d \ln Z_v}{d \ln \mu} = \frac{g^2}{8\pi^2}(\alpha - 3) + O(g^4) . \quad (26)$$

Using the relation between μ , g^2 and the asymptotic scale parameter $\Lambda_{\overline{MS}}$, we may rewrite Eq. (25) as

$$\ln \frac{v^2}{\Lambda_{\overline{MS}}^4} = \frac{16\pi^2}{g^2} \left(\frac{2}{\beta_0} - \frac{1}{\alpha} \right) + 2 + O(\ln g, g^2) , \quad (27)$$

where β_0 is the lowest order coefficient of the β -function of this model ($\beta_0 = (22 - 2n_f)/3$ with n_f quark flavors in the fundamental representation as matter). Apart from an anomalous dimension, the non-trivial solution v at sufficiently small coupling is thus proportional to the physical scale $\Lambda_{\overline{MS}}^2$ in the *particular* gauge $\alpha = \beta_0/2$. The anomalous dimension γ_v in Eq. (26) furthermore is of order g^4 at $\alpha = 3$. For $n_f = 2$ quark flavors, the terms of order $\ln g$ in Eq. (27) thus also vanish in the particular gauge $\alpha = \beta_0/2 = 3$ and higher order corrections to the asymptotic value of v at small g^2 are analytic in g^2 . With $n_f = 2$ flavors, one can expand the model about

$$v^2 = e^2 \Lambda_{\overline{MS}}^4 (n_f = 2) (1 + O(g^2)) \quad (28)$$

in the gauge $\alpha = \beta_0/2$, and determine the $O(g^2)$ corrections in Eq. (28) order by order in the loop expansion of the gap equation Eq. (20). Note that this behavior is surprisingly consistent with the previous observation⁴ for $SU(n)$ in generalized covariant gauges that the lowest order solution to the gap equation remains accurate to order g^2 at any finite order of the loop expansion in the gauge $\alpha = \beta_0/n$ when there are $n_f = n$ light quark flavors. This *does not* mean that other gauges are any less physical, but it does single out $\alpha = \beta_0/n = 3$ as a *critical* gauge in which the *perturbative* evaluation of the gap equation Eq. (20) is consistent for sufficiently small values of g^2 . (In QED the hydrogen spectrum to lowest order is most readily obtained in Coulomb gauge, although it evidently does not depend on the chosen gauge. In the present case asymptotic freedom determines an optimal gauge for solving the gap equation at small coupling.)

^dThis corrects the error in¹⁷ of ignoring the corrections of order g^2 in $Z = 1 + O(g^2)$

At the one-loop level, Eq. (20) has a unique non-trivial solution in any gauge $\alpha \neq 0$ and $\Gamma_{\sigma\sigma}$ of Eq. (24) shows that it corresponds to a minimum of the one-loop action. In the limit $\alpha \rightarrow 0$ at finite coupling, the non-trivial one-loop solution Eq. (25) coincides with the trivial one. On the other hand, some of the couplings in the non-linear gauge-fixing \mathcal{L}_{AG} become large in this limit, invalidating any perturbative analysis.

The highly singular behavior of the model when $\alpha \sim 0$ is already apparent in the divergent part of the W self-energy to one loop. The corresponding anomalous dimensions γ_W and γ_α of the vector boson and the gauge parameter are

$$\begin{aligned}\gamma_W &= -\frac{d \ln Z_W}{d \ln \mu} = \frac{g^2}{8\pi^2} \left(\beta_0 - \frac{9}{2} - \frac{\alpha}{2} - \xi \right) + O(g^4), \\ \gamma_\alpha &= \frac{d \ln \alpha}{d \ln \mu} = -\frac{g^2}{8\pi^2} \left(\frac{3}{\alpha} + 6 - \beta_0 + \alpha \right) + O(g^4).\end{aligned}\tag{29}$$

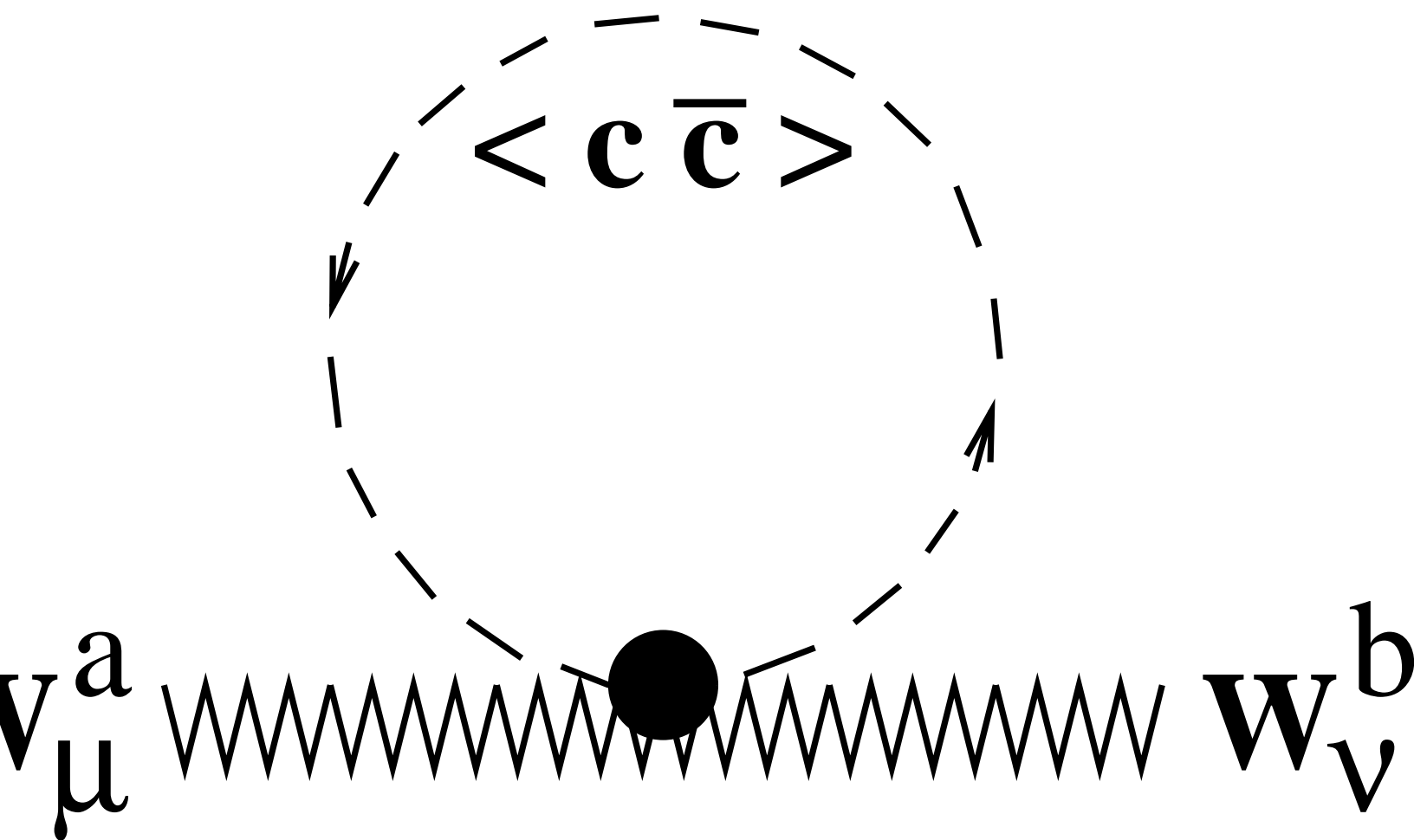
Gauge dependent interaction terms proportional to g/α at one loop thus lead to a term of order g^2/α^2 in the longitudinal part of the W self-energy only. The transverse part of the W self-energy is regular in the limit $\alpha \rightarrow 0$. Taking α to vanish thus is rather tricky: Eq. (29) implies that the longitudinal part of the W -propagator at one loop is proportional to $3g^2p^2 \ln(p^2)$ at large momenta and no longer vanishes in this limit. Higher order loop corrections similarly contribute to the longitudinal propagator as $\alpha \rightarrow 0$. γ_α does not depend on the gauge parameter ξ at one loop, due to an Abelian Ward identity that also gives the QED-like relation¹⁴ $Z_A = Z_g^{-2} = Z_\xi$ between the renormalization constants of the photon, of the coupling g and of the gauge parameter ξ .

The anomalous dimension of the gauge parameter α at sufficiently small g^2 is negative for positive values of α when $\beta_0 < 6 + 2\sqrt{3}$. With $\gamma_\alpha < 0$, the effective gauge parameter tends to decrease at higher renormalization scales μ and direct integration of Eq. (29) gives a vanishing α at a *finite* value of the coupling g^2 . As already noted above, the loop expansion, however, is valid only if $g^2 \ll 1$ and $g^2/\alpha \ll 1$. But Eq. (29) does show that there is no *finite* UV fixed point for the gauge parameter and that α effectively vanishes at least as fast as g^2 as $\mu \rightarrow \infty$ for any fixed gauge at finite g^2 . Eq. (28) nevertheless is the asymptotic solution to Eq. (20) in the sense that it is valid at arbitrary small coupling if the gauge at that coupling is chosen to be $\alpha(g) = \beta_0/2$.

The existence of a (unique) non-trivial solution to the gap equation can be viewed as a consequence of the scale anomaly⁴. The renormalization point dependence of Eq. (25) and the associated UV-divergence of the loop integral are an indication of this.

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